

**Cardinality and Counting**

The cardinal value of a number refers to the quantity of things it represents, e.g. the numerosity, ‘how manyness’, or ‘threeness’ of three. When children understand the cardinality of numbers, they know what the numbers mean in terms of knowing how many things they refer to. Counting is one way of establishing how many things are in a group, because the last number you say tells you how many there are. Children enjoy learning the sequence of counting numbers long before they understand the cardinal values of the numbers. Subitising is another way of recognising how many there are, without counting.

<p><b>Counting: saying number words in sequence</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need to know number names, initially to five, then ten, and extending to larger numbers, including crossing boundaries 19/20 and 29/30.</p> <p>Counting back is a useful skill, but young children will find this harder because of the demand it places on the working memory.</p>	<ul style="list-style-type: none"> <li>● counting backwards, for example <i>number rhymes</i></li> <li>● starting from different numbers.</li> </ul>
<p><b>Counting: tagging each object with one number word</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need lots of opportunities to count things in irregular arrangements. For example, how many play people are in the sandpit? How many cars have we got in the garage? These opportunities can also include counting things that cannot be seen, touched or moved.</p>	<ul style="list-style-type: none"> <li>● counting things of different sizes – this helps children to focus on the numerosity of the count</li> <li>● counting things that can’t be seen, such as sounds, actions, words</li> <li>● counting things that cannot be moved, such as pictures on a screen, birds at the bird table, faces on a shape.</li> </ul>
<p><b>Counting knowing the last number counted gives the total so far</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need the opportunity to count out or ‘give’ a number of things from a larger group, not just to count the number that are there. This is to support them in focusing on the ‘stopping number’ which gives the cardinal value.</p>	<ul style="list-style-type: none"> <li>● playing dice games to collect a number of things</li> <li>● playing track games and counting along the track.</li> </ul>
<p><b>Subitising: recognising small quantities without needing to count them all</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Subitising is recognising how many things are in a group without having to count them one by one. Children need opportunities to see regular arrangements of small quantities, e.g. a dice face, structured manipulatives, etc., and be encouraged to say the quantity represented. Children also need opportunities to recognise small amounts (up to five) when they are not in the ‘regular’ arrangement, e.g. small handfuls of objects.</p>	<ul style="list-style-type: none"> <li>● using dot cards, dominoes and dice as part of a game, including irregularly arranged dots (e.g. stuck on)</li> <li>● playing hidden object games where objects are revealed for a few seconds; for example, small toys hidden under bowl – shuffle them, lift the bowl briefly and ask how many there were</li> <li>● ‘all at once fingers’ – show me four fingers.</li> </ul>
<p><b>Numeral meanings</b></p>	<p><b>Activities &amp; opportunities</b></p>

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<p>Children need to have the opportunity to match a number symbol with a number of things. Look for opportunities to have a range of number symbols available, e.g. wooden numerals, calculators, handwritten (include different examples of a number, e.g. 4, 4, 4).</p>	<ul style="list-style-type: none"> <li>● using numeral dice in games; matching numerals with varied groups of things</li> <li>● using 'tidy-up labels' on containers and checking that nothing is missing</li> <li>● reading number books</li> <li>● putting the right number of snacks on a tray for the number of children shown on a card.</li> </ul>
<p><b>Conservation: knowing that the number does not change if things are rearranged (as long as none have been added or taken away)</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need the opportunity to recognise amounts that have been rearranged and to generalise that, if nothing has been added or taken away, then the amount is the same.</p>	<ul style="list-style-type: none"> <li>● correcting a puppet who may say that there are more or fewer objects now, as they have been moved around, e.g. spread out or pushed together</li> <li>● contexts such as sharing things out (grouping them in different ways) and then the puppet complaining that it is not fair as they have less</li> <li>● encouraging the children to make different patterns with a given number of things.</li> </ul>
<p><b>Common errors in this area:</b></p>	<p><b>What to look for, can a child:</b></p>
<ul style="list-style-type: none"> <li>● missing out an object or counting an object twice</li> <li>● when asked how many cars are in a group of four, simply recounting '1, 2, 3, 4,' without concluding that 'there are four cars in the group'</li> <li>● when asked to 'get five oranges' from a tray, a child just grabs some, or carries on counting past five</li> <li>● when objects in a group are rearranged, the child (unnecessarily) recounts them to find how many there are</li> <li>● difficulties in counting back</li> <li>● confusion over the 'teen' numbers – they are hard to learn</li> <li>● missing a number like 15 (13 or 15 are commonly missed out) or confusing 'thirteen' and 'thirty'.</li> </ul>	<ul style="list-style-type: none"> <li>● consistently recite the correct sequence of numbers and cross decade boundaries?</li> <li>● collect nine from a large pile, e.g. nine pencils from a pot?</li> <li>● subitise (instantly recognise) a group that contains up to four, then five, in a range of ways, e.g. fingers, dice, random arrangement?</li> <li>● select a numeral to represent a quantity in a range of fonts, e.g. 4, 4, 4?</li> <li>● correct a puppet who thinks the amount has changed when their collection has been rearranged?</li> </ul>

## Progression in Early Maths

### Comparison

Comparing numbers involves knowing which numbers are worth more or less than each other. This depends both on understanding cardinal values of numbers and also knowing that the later counting numbers are worth more (because the next number is always one more). This understanding underpins the mental number line which children will develop later, which represents the relative value of numbers, i.e. how much bigger or smaller they are than each other.

More than/less than	Activities & opportunities
Children need progressive experiences where they can compare collections and begin to talk about which group has more things. Initially, the groups need to be very obviously different, with one group having a widely different number of things. Collections should also offer challenges, such as including more small things and fewer large things, to draw attention to the numerosity of the comparison, i.e. the number of things, not the size of them.	<ul style="list-style-type: none"> <li>● collections for children to sort and compare, which include objects which are identical, and which include objects of different kinds or sizes</li> <li>● collections with a large number of things, and collections with a small number of things.</li> </ul>
Identifying groups with the same number of things	Activities & opportunities
Children need the opportunity to see that groups could consist of equal numbers of things. Children can check that groups are equal, by matching objects on a one-to-one basis.	<ul style="list-style-type: none"> <li>● ensuring that when providing groups to compare, there are some that have an equal amount</li> <li>● asking children to convert two unequal groups into two that have the same number, e.g. 'There are 6 apples in one bag and 2 in another bag; can we make the bags equal for the two hungry horses?'</li> </ul>
Comparing numbers and reasoning	Activities & opportunities
<p>Children need opportunities to apply their understanding by comparing actual numbers and explaining which is more. For example, a child is shown two boxes and told one has 5 sweets in and the other has 3 sweets in. Which box would they pick to keep and why? Look for the reasoning in the response they give, i.e. 'I would pick the 5 box because 5 is more than 3 and I want more.' If shown two numerals, children can say which is larger by counting or matching one-to-one.</p> <p>Children can compare numbers that are far apart, near to and next to each other. For example, 8 is a lot bigger than 2 but 3 is only a little bit bigger than 2.</p>	<ul style="list-style-type: none"> <li>● explaining unfair sharing - 'This one has more because it has 5 and that one only has 3'</li> <li>● comparing numbers that are far apart, near to, and next to each other.</li> </ul>
Knowing the 'one more than/one less than' relationship between counting numbers	Activities & opportunities
<p>Children need opportunities to see and begin to generalise the 'one more than/one less than' relationship between sequential numbers. They can apply this understanding by recognising when the quantity does not match the number, i.e. if a pack is labelled as 5 but contains only 4, the children can identify that this is not right.</p> <p>Support children in recognising that if they add one, they will get the next number, or if one is taken away, they will have the previous number. For example: 'There are 4 frogs on the log, 1 frog jumps off. How many will be left? How do you know?'</p>	<ul style="list-style-type: none"> <li>● labelling groups with the correct numeral. Do children spot the error if a group is mislabelled? For example, 'The label on the pot says 4 and we have 5 – what do we need to do?' A child may say, 'We need to take one out because we have one too many.'</li> <li>● ensuring children focus on the numerosity of the group by having items in the collection of different kinds and sizes</li> </ul>

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	<ul style="list-style-type: none"> <li>making predictions about what the outcome will be in stories, rhymes and songs if one is added to, or if one is taken away.</li> </ul>
<b>Common errors in this area:</b>	<b>What to look for, can a child:</b>
<ul style="list-style-type: none"> <li>children not comparing the numerosity of the group and considering more in terms of size</li> <li>children giving a response that does not match the context when estimating a number; e.g. when adding, giving as an answer a number that is smaller than the numbers given. Example: 'There are 7 cars in a garage and then 2 more go in.' The child guesses there are 4 cars in total inside.</li> </ul>	<ul style="list-style-type: none"> <li>state which group of objects has more? Can they do this with a large or small visual difference?</li> <li>compare two numbers and say which is the larger?</li> <li>predict how many there will be if you add or take away one?</li> </ul>

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### Composition

Knowing numbers are made up of two or more other smaller numbers involves 'part-whole' understanding. Learning to 'see' a whole number and its parts at the same time is a key development in children's number understanding. Partitioning numbers into other numbers and putting them back together again underpins understanding of addition and subtraction as inverse operations.

Part-whole: identifying smaller numbers within a number (conceptual subitising – seeing groups and combining to a total)	Activities & opportunities
<p>Children need opportunities to see small numbers within a larger collection. 'Number talks' allow children to discuss what they see. For instance, with giant ladybirds: 'There are 5 spots altogether. I can see 4 and 1, I can see 3 and 2, and I can see 1 and 1 and 1 and 1 and 1.'</p> <p>Encourage exploration of all the ways that 'five' can be and look. Children are encouraged to look closely at numbers to see what else they can see. This reinforces the concept of conservation.</p>	<p>encouraging making arrangements with (e.g.) ten; ensuring the children talk about the different arrangements they can see within the whole.</p>
Inverse operations	Activities & opportunities
<p>Children need opportunities to partition a number of things into two groups, and to recognise that those groups can be recombined to make the same total. Encourage children to say the whole number that the 'parts' make altogether.</p>	<p>exploring songs; for example, 'Five Currant Buns' – show that the whole is still five, but some are in the shop and some have been taken away; check throughout that there are still five currant buns</p> <p>playing skittles and looking at how many are standing. How many have fallen over? How many are there altogether?</p>
A number can be partitioned into different pairs of numbers	Activities & opportunities
<p>Children need opportunities to explore a range of ways to partition a whole number. The emphasis here is on identifying the pairs of numbers that make a total. Children can do this in two ways – physically separating a group, or constructing a group from two kinds of things.</p>	<p>Numicon towers: layering up Numicon pieces of the same total</p> <p>putting things into two containers in different ways</p> <p>making a number with two different kinds of things. For example, make a fruit skewer with five pieces of fruit, using bowls of bananas/strawberries to choose from; then ask the children to describe how they have made theirs. They should compare it with a partner's: 'What is the same about your skewers? What is different?'</p> <p>Bunny Ears: using your fingers like bunny ears. 'With two hands, show me five fingers. Can you do it in a different way?' Or, 'Show five fingers altogether with a friend.'</p> <p>Spill the Beans: using double-sided counters or beans, where one side is coloured, throw the collection and note how many of each type can be seen and how many altogether.</p>

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	using six bean bags with different fabric on each side, throw the collection and note how many of each type can be seen.
<b>A number can be partitioned into more than two numbers</b>	<b>Activities &amp; opportunities</b>
Children need opportunities to explore the different ways that numbers can be partitioned, i.e. into more than two groups. Situations to promote this include increasing the number of pots to put a given amount into, e.g. planting ten seeds into three or more pots.	<p>role play, e.g. in a toy shop, ten toys need arranging onto the three shelves. How will you organise them?</p> <p>having more than two places to sort things into in any given context, e.g. arranging characters in small-world play in different locations</p> <p>games such as 'Posh Ducks' (Griffiths, R., Back, J. &amp; Gifford, S. (2016) Making Numbers: Using manipulatives to teach arithmetic, OUP): using a set number of ducks, for example ten in three different locations (nest, water, decking), roll the dice and make one group match the amount shown without adding or taking any away.</p>
<b>Number bonds: knowing which pairs make a given number</b>	<b>Activities &amp; opportunities</b>
Children need opportunities to say how many are hidden in a known number of things. For example: 'Five toys go into a tent, then two come out. How many are left in the tent?' The child should respond that there are still three toys in the tent.	<p>playing hiding games with a number of objects in a box, under a cloth, in a tent, in a cave, etc.</p> <p>utilising classroom routines such as tidy-up time to identify how many are still missing from a pot with a number label.</p>
<b>Common errors in this area:</b>	<b>What to look for, can a child:</b>
children suggesting that a larger number than the total are hidden.	<p>subitise small groups within a larger number?</p> <p>make a reasonable guess at a hidden number?</p> <p>in context, state two groups that make a larger amount? For example, how might the (six) bean bags land? You could have three with stripes up and three with spots up.</p>

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### Pattern

Seeking and exploring patterns is at the heart of mathematics (Schoenfeld, 1992). Developing an awareness of pattern helps young children to notice and understand mathematical relationships. Clements and Sarama (2007) identify that patterns may provide the foundations of algebraic thinking, since they provide the opportunity for young children to observe and verbalise generalisations.

The focus in this section is on repeating patterns, progressing from children copying simple alternating AB patterns to identifying different structures in the 'unit of repeat', such as ABB or ABBC. Patterns can be made with objects like coloured cubes, small toys, buttons and keys, and with outdoor materials like pine cones, leaves or large blocks, as well as with movements and sounds, linking with music, dance, phonics and rhymes. Children can also spot and create patterns in a range of other contexts, such as printed patterns, timetables, numbers and stories.

Counting an AB pattern	Activities & opportunities
Children need the opportunity to see a pattern, to talk about what they can see, and to continue a pattern. At first, they will do this one item at a time, e.g. red cube, blue cube, red cube...verbalising the pattern helps. Children may then be asked to say what they would add next to continue it.	<ul style="list-style-type: none"> <li>● building towers or trains of different-coloured cubes (continuing patterns horizontally and vertically)</li> <li>● extending patterns using a wide range of identical objects in different colours, e.g. beads; small plastic toys such as bears, dinosaurs, vehicles. Try to avoid interlocking cubes or bead-threading so children can focus on the pattern rather than their coordination skills.</li> </ul>
Copying an AB pattern	Activities & opportunities
Copying a pattern can be difficult for children if they have to keep comparing item by item. AB patterns are easiest – when presented to children, these should contain several repeats, to ensure that the pattern unit is evident. Discuss the nature of the pattern: how has the pattern been made? Patterns can have a range of features such as varying objects, size or orientation.	<ul style="list-style-type: none"> <li>● accessing a range of patterns to copy. For example, using the plastic bears: big, small, big, small, big... footwear: shoe, welly, shoe, welly..., actions and sounds: jump, twirl, jump, twirl, jump... or clap, stamp, clap, stamp...</li> <li>● collecting things in the outdoors environment: leaf, stick, leaf, stick...</li> </ul>
Make their own AB pattern	Activities & opportunities
<p>As children progress from continuing to copying patterns, they can be challenged to change the sample pattern or to create their own. A range of objects can be provided for children to decide what the features of the pattern are going to be. Children may find it easier to make a pattern with the same colours as the original but with different objects. For example, copying a red–blue cube pattern with red and blue dinosaurs is easier than with yellow and green cubes. Patterns can involve different aspects and modes such as sounds, words or actions: some children will need suggestions, while others will think of their own. As children construct the patterns, ensure they have opportunities to:</p> <ul style="list-style-type: none"> <li>● repeat the unit at least three times (big bear, small bear; big bear, small bear; big bear, small bear). This is to ensure the child can sustain the pattern</li> <li>● make a specified pattern, e.g. 'Can you do a green, yellow pattern?' This is to ensure the child can apply their pattern understanding</li> <li>● choose their own rule, e.g. 'I am going to make a big, small pattern.' This is to ensure the child can identify pattern features/rules/criteria</li> </ul>	<ul style="list-style-type: none"> <li>● challenging the child to change one element of the pattern they have created, e.g. 'Can you change the red bear to a blue bear? What is the pattern now?'</li> <li>● ensuring that there are numerous opportunities to create patterns – e.g. in the outdoors, using natural materials such as sticks, leaves, stones, pine cones; in craft activities, using stamping, sticking, printing; with musical instruments, using sounds such as drums, shakers, triangles, etc.</li> <li>● working collaboratively with a friend to take turns to create a pattern, e.g. one claps, one stamps, or one gets the red bear, one gets the yellow bear, etc.</li> <li>● challenging a friend to continue or copy their pattern.</li> </ul>

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<ul style="list-style-type: none"> <li>choose their own actions or sounds, e.g. clap, stamp... This is to help children generalise the idea of pattern.</li> </ul>	
<p><b>Spotting an error in an AB pattern</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>When working with AB patterns, children also need the opportunities to spot and correct errors. It is easiest to spot an extra item, then a missing item, then items swapped around. When presented with an AB pattern, children can be encouraged to describe it to make sure it is right. Then, to detect an error, they can track the pattern from the start. To begin with, children may know there is something wrong, but might not be able to say what the error is. They then might take several attempts to correct it, before being able to repair the error in one move.</p>	<ul style="list-style-type: none"> <li>presenting patterns with deliberate errors, including extra, missing and swapped items, e.g. red cube, blue cube, red cube, blue cube, red cube, red cube, blue cube – identifying there is an extra item and fixing it by removing the extra red cube, putting in an extra blue cube, or swapping the final cubes</li> <li>asking the children to make a pattern with a deliberate mistake and challenging a friend to spot it.</li> </ul>
<p><b>Identifying the unit of repeat</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>The key aspect of understanding patterns is identifying the smallest part of the pattern, or the 'unit of repeat' You can draw children's attention to this when building patterns by picking up a unit at a time, e.g. a blue block and a red block together, and describing this as a 'red- blue pattern', rather than a red, blue, red, blue, red, blue pattern. Children can also be asked to show the pattern unit which repeats, e.g. show two blocks, a red and a blue</p>	<ul style="list-style-type: none"> <li>highlight within a pattern what the unit of repeat is and ask the children to describe it. At this point for pattern novices (children who aren't as experienced as others), it would be good to do this with physical objects so that the unit of repeat can be moved to show how it repeats. Patterns that are printed, stamped or stuck down, and therefore cannot be corrected, are more appropriate for more confident pattern makers.</li> </ul>
<p><b>Continuing an ABC pattern</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Once children are secure with alternating patterns, they can tackle more complex pattern structures:</p> <p>ABC has more items in the unit of repeat, but all different, e.g. red, blue, yellow; red, blue, yellow...</p> <p>ABB is more challenging because they have two items within the same unit of repeat, e.g. red, blue, blue; red, blue, blue...</p> <p>ABBC is more complex because it is longer, with three items, but also includes items which are the same, e.g. red, blue, blue, yellow; red, blue, blue, yellow...</p> <p>AABB may be simpler as there are just two items, both repeated, e.g. red, red, blue, blue; red, red, blue, blue...</p> <p>Children who have only experienced alternating ABC patterns may state that patterns such as ABBC are not patterns, as you cannot have two of the same colour next to each other. This highlights that children need lots of experience of a range of pattern types, so early misconceptions do not form about what makes a pattern. When working on continuing these patterns, children should be encouraged to focus on the unit of repeat, e.g. 'I see you are making a red, blue, green pattern'. Ensure that children repeat the</p>	<ul style="list-style-type: none"> <li>building towers or trains of different-coloured cubes (continuing patterns horizontally and vertically)</li> <li>extending patterns using a wide range of identical objects in different colours, e.g. beads; small plastic toys such as bears, dinosaurs and vehicles.</li> </ul> <p>Try to avoid using interlocking cubes or bead-threading, so children can focus on the pattern they are constructing rather than on their coordination skills.</p>



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<p>pattern at least three times and are encouraged to describe and say how they would continue.</p>	
<p><b>Continuing a pattern which ends mid-unit</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>As children work on patterns involving more elements, they can be challenged to continue patterns which do not end after a whole unit of repeat. Provide experiences where the given pattern stops mid-unit.</p>	<ul style="list-style-type: none"> <li>● providing a range of patterns – physical and on cards – that children can continue</li> <li>● ensuring that the patterns offered have different structures and end after a complete or a partial unit.</li> </ul>
<p><b>Make their own ABB, ABBC patterns</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>As with the first stages of making an AB pattern, the same range of experiences needs to be provided when the unit of repeat extends. A range of objects can be provided for children to decide what the features of the pattern are going to be. Patterns may include varied items and modes, such as sounds and actions. Ensure that children have opportunities to:</p> <ul style="list-style-type: none"> <li>● repeat the unit at least three times (big bear, small bear, medium bear; big bear, small bear, medium bear; big bear, small bear, medium bear). This is to ensure the pattern can be sustained over a longer duration</li> <li>● make a specified pattern, e.g. ‘Can you do a green, yellow, blue pattern?’ This is to ensure the child can apply their pattern understanding</li> <li>● choose their own rule, e.g. ‘I am going to make a big, small, small pattern.’ This is to ensure the child can identify pattern features/rules/criteria</li> <li>● choose their own actions or sounds, e.g. clap, stamp, twirl... This is to support children in generalising pattern structures.</li> </ul>	<ul style="list-style-type: none"> <li>● utilising a range of items in the environment to create patterns such as interlocking cubes and toys, e.g. links, elephants, camels</li> <li>● exploring and creating patterns on peg boards, with fruit (e.g. fruit kebabs), musical instruments, movements and dance sequences.</li> </ul>
<p><b>Spotting an error in an ABB pattern</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>When working with ABB patterns, children also need the opportunities to spot and correct errors. It is easiest to spot an extra item, then a missing item, then items swapped around. When presented with an ABB pattern, children can be encouraged to describe it to make sure it is right. Then, to detect an error, they can track the pattern from the start. To begin with, children may know there is something wrong, but might not be able to say what the error is. They then might take several attempts to correct it, before being able to repair the error in one move.</p>	<ul style="list-style-type: none"> <li>● presenting patterns with deliberate errors</li> <li>● once children have fixed the pattern, encouraging them to check the ‘fix’ by tracking the pattern</li> <li>● asking the children to make a pattern with a deliberate mistake and challenging a friend to spot it.</li> </ul>
<p><b>Symbolising the unit structure</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>As children become more experienced with pattern- continuing, -extending and -creating, encourage them to record the patterns that they make. Initially this might be straightforward representations, but over time these recordings may become more</p>	<ul style="list-style-type: none"> <li>● including the following phrasing in discussion and dialogue: ‘This is a red blue pattern; this/that; I call it an A (one of these) then a B (one of those).’</li> </ul>

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iconic, e.g. a red dot representing the red dinosaur, a squiggle or the letter R for red dinosaur. As this progresses, encourage the children to symbolise their patterns in a range of ways, and to use these symbols to continue the pattern to demonstrate their understanding of the pattern. Children may, with adult direction, pick up on the coding of patterns as AB, ABB, ABBC, etc. One additional level of challenge is to create symbols for movement/sound patterns, as the children need to construct a symbol with less concrete/visual support.

- constructing patterns with actions and developing symbols to show the pattern and to provide 'instructions' for someone else to follow the pattern
- inviting friends to copy the pattern from the symbols.

### Generalising structures to another context or mode

As children gain experience of symbolising patterns, they develop their experience of pattern structure. As they identify the unit of repeat and express it, they will be able to use this knowledge to create a pattern in a different medium, which follows the same structure.

For example, a child might be working with a pattern like this:



You may ask them to describe the pattern, what comes next, what the rule is for their pattern, etc. If a child can do this confidently, they could be asked to recreate the same pattern rule with different objects.

'Can you use the nature basket to create a pattern with the same rule?'

The child would need to recognise they need three different items, one of which is duplicated. They may say they will use a twig instead of the circle, a leaf instead of the square, a conker instead of the triangle, and create this instead:



### Activities & opportunities

- providing a range of experiences where children can create a pattern using a coding structure
- ensuring children can follow the patterns they have coded.

### Making a pattern which repeats around a circle

As children become more experienced with the structures of patterns, they can investigate whether patterns can continue indefinitely in a circle. Linking elephants,

### Activities & opportunities

- making circular patterns such as necklaces, circles of linking elephants or camels

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camels or making a necklace can provoke discussion about this. You then might lead discussions about whether the pattern works and how you can tell. If it doesn't work, can children explain why, and correct it so it does? Circles allow children to adjust the circle size, so they can add or take out items.

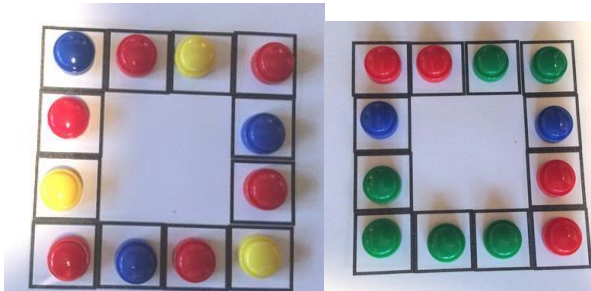


- using pre-given circles to create a border, such as on or around a paper plate
- exploring which patterns work, which don't, and why
- offering a unit of the pattern and asking the child if they can include it in their pattern
- making patterns around rectangular or other shaped frames.

### Making a pattern around a border with a fixed number of spaces

This is where the children explore creating a pattern around a given space. In these sorts of activities, children have the additional challenge of recognising if their pattern can 'work' – fit into the given space. It is useful to include indoor and outdoor spaces, e.g. creating an outdoor reading area and defining it with a border of carpet tiles. Children can create a pattern on the carpet tiles with cubes to see if their pattern works, e.g. one coloured cube per tile.

A pattern that works: A pattern that doesn't work:



### Activities & opportunities

- creating borders around defined spaces in the learning environment, i.e. a garden for the teddy bears, an outdoor reading area, etc.
- encouraging children to predict if the pattern could 'keep going', voting on this and discussing their thoughts and reasons with a partner.

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<p>When exploring if a pattern works or not, draw attention to the number of spaces and the size of the unit of repeat.</p>	
<p><b>Pattern spotting around us</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>As children become pattern experts, look for opportunities to spot and study patterns in the environment. These patterns could be in construction, fabric, wrapping paper, wallpaper, etc. Look for opportunities to identify the unit of repeat and explain how it repeats.</p> <p>Consider other patterns, such as growing patterns, extending a cross shape, or spotting 'staircase' patterns of numbers going up in ones or twos. Children may make and spot spatial patterns, for example reflecting shapes or reversing an image.</p> <p>Stories and rhymes present a good opportunity to explore a growing pattern, e.g. 'There was an Old Lady who Swallowed a Fly', or 'A Squash and a Squeeze'. Explore representing these diagrammatically – to see a staircase pattern, for example.</p>	<ul style="list-style-type: none"> <li>● exploring patterns in stories, songs and rhymes</li> <li>● where possible, representing these diagrammatically to support pattern-spotting, and predicting what will happen next, and why</li> <li>● inviting children to spot patterns in the home environment, or bring in examples from home</li> <li>● looking at fabric patterns from different cultural traditions: discussing the patterns in terms of what stays the same and what is different</li> <li>● designing wrapping paper for a specific event that involves creating a pattern which the children can describe.</li> </ul>
<p><b>Common errors in this area:</b></p>	<p><b>What to look for, can a child:</b></p>
<ul style="list-style-type: none"> <li>● not recognising a pattern such as ABBA (e.g. stating that patterns cannot have two of the same colour together)</li> <li>● when copying or extending a pattern, changing it before making three repeats</li> <li>● spotting that there is an error but not being able to describe it</li> <li>● identifying an error but not being able to correct it</li> <li>● correcting an error by making a 'local correction', which just moves the problem along (e.g. by adding an extra item when colours have been swapped)</li> <li>● describing the whole pattern instead of identifying the part which repeats, or the unit of repeat.</li> </ul>	<ul style="list-style-type: none"> <li>● continue, copy and create an AB pattern?</li> <li>● identify the pattern rule (unit of repeat) in an AB pattern?</li> <li>● continue, copy and create ABB, ABBC (etc.) patterns?</li> <li>● identify the pattern rule (unit of repeat) in ABB, ABBC (etc.) patterns?</li> <li>● spot an error and 'correct' a pattern?</li> <li>● explain whether a circular pattern is continuous or not?</li> </ul>

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### Shape and Space


Mathematically, the areas of shape and space are about developing visualising skills and understanding relationships, such as the effects of movement and combining shapes together, rather than just knowing vocabulary. Spatial skills are important for understanding other areas of maths and children need structured experiences to ensure they develop these. Here, the focus is on actively exploring spatial relations and the properties of shapes, in order to develop mathematical thinking (rather than on shape classification, which requires prior knowledge of properties). This section is concerned with developing the two aspects of spatial awareness and shape awareness, with some progression identified within each.

Developing spatial awareness: experiencing different viewpoints	Activities & opportunities
<p>Children need opportunities to move both themselves and objects around, so they see things from different perspectives. This will support them in visualising how things will appear when turned around and imagining how things might fit together. They need to make constructions, patterns and pictures, and select shapes which will fit when rotated or flipped in insert boards, shape sorters and jigsaws. These experiences will support them in noticing the results of rotating and reflecting images, and in visualising these.</p>	<ul style="list-style-type: none"> <li>● riding trikes around interesting routes</li> <li>● construction activities</li> <li>● printing and making pictures and patterns with shapes</li> <li>● posting boxes</li> <li>● jigsaws</li> <li>● making a complete circuit with a train track</li> <li>● directing a simple robot or remote-controlled toy vehicle along a route</li> <li>● tangrams: 'Can you make a person with the shapes?'</li> <li>● with toys in a line: 'Can you say what the teddy on the other side is seeing?'</li> </ul>
Developing spatial vocabulary	Activities & opportunities
<p>Children need opportunities to be exposed to and to use the language of position and direction: position: 'in', 'on', 'under' direction: 'up', 'down', 'across'. Children also need opportunities to use terms which are relative to the viewpoint: 'in front of', 'behind', 'forwards', 'backwards' ('left' and 'right' to be used later on as ideas develop). Create as many opportunities as possible to explore this language, taking advantage of play in the outdoors to explore sequences of body movements (following obstacle courses, directing a friend, etc.).</p>	<ul style="list-style-type: none"> <li>● hunting for hidden objects, with some prompts, e.g. 'Look behind the bicycle store, take three steps from the front of the art cupboard...'</li> <li>● developing and talking about small-world scenarios, e.g. doll's house, miniature village, play park</li> <li>● acting out their own versions of well-known stories where characters negotiate routes and obstacles, for example 'We're Going on a Bear Hunt'</li> <li>● directing each other as robots.</li> </ul>
Shape awareness: developing shape awareness through construction	Activities & opportunities
<p>Through play – particularly in construction – children have lots of opportunities to explore shapes, the attributes of particular shapes, and to select shapes to fulfil a particular need. Support this exploration by discussing items built by children in terms of how towers are built and why certain shapes are chosen to make a tower, and the space that has been created within an enclosure. Ask: 'How did you make that tower?', 'Why were those blocks good ones to use?'</p>	<ul style="list-style-type: none"> <li>● construction with structured and unstructured materials</li> <li>● making dens with varied materials outdoors.</li> </ul>
Representing spatial relationships	Activities & opportunities
<p>Small world play and model building provide lots of opportunities for children to describe things being 'in front of', 'behind', 'on top of' etc., and to consider objects from different</p>	<ul style="list-style-type: none"> <li>● designing a plan for a garden or play area, using a small tray with sand, twigs, building bricks, etc</li> </ul>

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<p>perspectives. Drawing representations of these relationships is a further challenge. These drawings may include a simple representation of a three-dimensional object from a different viewpoint. For example, 'can you draw your construction from above, looking down on it?'</p>	<ul style="list-style-type: none"> <li>● drawing or making a simple map of a route with 'landmarks', e.g. houses and trees</li> <li>● following a simple map of an excursion.</li> </ul>
<p><b>Identifying similarities between shapes</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need opportunities to construct and create things that represent objects in their environment. As they do this, they should notice shape properties of the object that they want to represent; encourage them to think about the appropriateness of the shapes they choose. Examples of this may include representing a ball as a circle, building a train from wooden rectangular blocks, or using a curved block for the elephant's trunk.</p>	<ul style="list-style-type: none"> <li>● stories as a prompt for creating representations, e.g. building a house for the three bears</li> <li>● making pictures with found materials, as well as structured shapes and blocks.</li> </ul>
<p><b>Showing awareness of properties of shape</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>At this stage, children show increasing intentionality in their selection of shapes, for example using cylinders to represent wheels because they can roll. Draw children's attention to specific properties by using specific language in everyday situations, while children may use informal language. Properties may include:</p> <ul style="list-style-type: none"> <li>● curvedness</li> <li>● numbers of sides and corners (2D) or edges, faces and vertices (3D)</li> <li>● equal sides</li> <li>● parallel sides</li> <li>● angle size, including right angles</li> <li>● 2D shapes as faces of 3D shapes.</li> </ul> <p>In play, children show that they are utilising this knowledge by gathering specific items that are needed for their construction, e.g. making a bed for a teddy and gathering blocks of equal length to make the rectangle; taking time with constructing corners so the shapes fit together to make a right angle.</p>	<ul style="list-style-type: none"> <li>● making an insect hotel – selecting tube-like shapes from a collection of varied materials, some not fit for purpose</li> <li>● creating an extended channel for water to flow from a high container to a low one, some distance away</li> <li>● asking questions, for example: 'What shapes can you make with three people inside a loop of string? What about with four people?' 'What is the same and what is different about these?'</li> <li>● making shapes with sticks and with their own bodies</li> <li>● printing with shapes: 'What footprint do you think this cylinder will make? What about if you roll it?'</li> </ul>
<p><b>Describing properties of shape</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>As children construct, and appear to be utilising, the properties of shapes, informally ask them about their constructions and representations. Children may use comparisons such as 'ball-shaped' or 'house-shaped', or start to discriminate between shapes, e.g. a 'fat' triangle and a 'pointy' triangle, using informal language.</p> <p>With shapes such as triangles and rectangles, ensure that children are used to seeing a range of examples, and the same shape in different orientations, as well as different sizes, colours and materials.</p>	<ul style="list-style-type: none"> <li>● covering objects in foil and inviting children to justify their guesses about what is inside</li> <li>● making arrangements with a selection of different rectangles, including squares.</li> </ul>

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Developing an awareness of relationships between shapes	Activities & opportunities
<p>As children become more confident with specific shapes, encourage them to spot shapes within shapes. You might talk about small triangles making a bigger triangle or identifying 2D faces of 3D shapes. Pattern blocks are a useful resource, since children can point out the shapes they have used to make their whole pattern:</p>  <p>Also encourage children to predict what will happen when paper is cut or folded, or shapes are combined. Ask: 'What shapes will we see?', 'What will happen if we fold the square'</p>	<ul style="list-style-type: none"> <li>• choosing 2D shapes to construct a 3D model, e.g. using triangles and rectangles to make a tent</li> <li>• making decorations by folding and cutting</li> <li>• making 3D shapes using interlocking shapes.</li> </ul>
Common errors in this area:	What to look for, can a child:
<ul style="list-style-type: none"> <li>• children thinking that only regular triangles are triangles, only brick-like rectangles are rectangles (i.e. shapes are defined by their image, not by their properties)</li> <li>• children thinking that squares are only squares when the bottom is horizontal (i.e. shapes are defined by their orientation).</li> </ul>	<ul style="list-style-type: none"> <li>• select and rotate shapes to fit into a given space?</li> <li>• use positional vocabulary, including relative terms, to describe where things are in small-world play?</li> <li>• show intentionality in selecting shapes for a purpose, such as cylinders to roll?</li> <li>• make a range of constructions, including enclosures, and talk about the decisions they have made?</li> <li>• see shapes in different orientations and recognise that they are still that shape?</li> <li>• recognise a range of triangles and say how they know what they are?</li> </ul>

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### Measures

Mathematically, measuring is based on the idea of using numbers of units in order to compare attributes, such as length or capacity. Although young children engage with using rulers and experience being measured in centimetres, kilos – and years! – the measuring units themselves are hard to understand. Children need to realise which attribute is being measured, e.g. weight as opposed to size, and the idea of conservation: that the amount stays the same, even if the appearance alters, e.g. if dough is stretched out or in bits. In order to understand units, they need to realise that two items can be compared using a third item, or ‘go between’, such as a stick. Finally, children need to understand how equal size units are used repeatedly to express an amount as a number. While young children can engage actively in making comparisons and exploring equivalence of length, volume, capacity and weight in different ways, some of these ideas are challenging and will develop later in primary school. For instance, weight (mass or density) is difficult to distinguish from size since it is invisible, and the concept of conservation is harder to understand for weight and capacity. Measuring with non-standard units of different sizes in order to appreciate the need for equal units is less effective with younger children, so centimetre cubes are recommended as accessible units. While time is also elusive to measure, young children can sequence events and, for example, count ‘sleeps’. (Money as a measure of value is too advanced to consider here.)

Recognising attributes	Activities & opportunities
<p>In this first stage, children are able to recognise the specific attributes of (for example) length – that a stick is long; adults are tall. Their initial recognition may be a descriptor and over-applied (all straight things are long, and if it is not straight it cannot be long; all adults are tall).</p> <p>Children may use gestures or words to start to compare amounts of continuous quantities (length, capacity, weight), pointing to items that are big, tall, full or heavy. Children learn this vocabulary from the adults around them. Adults can seek opportunities to extend and refine conversations about things that are long, tall, high, heavy, full, etc. rather than just ‘big’. At this point children may not be using comparative language such as, ‘You are taller than me.’</p>	<ul style="list-style-type: none"> <li>● ensuring adults model language which highlights the specific attribute that is the focus of attention</li> <li>● dough modelling, which can provide a good opportunity to discuss the length of snakes, or the weight of different-sized lumps</li> <li>● water and sand-play, which can provide lots of opportunities to highlight capacity.</li> </ul>
Comparing amounts of continuous quantities	Activities & opportunities
<p>Children can find something that is longer/shorter or heavier/lighter than a given reference item. They will utilise strategies such as direct comparison, e.g. placing objects side by side to determine which is longer. Children compare sizes, lengths, weights and capacities verbally and begin to use more specific terms, such as ‘taller than’, ‘heavier than’, ‘lighter than’, and ‘holds more than’, as well as more general comparative phrases, such as ‘not enough’, ‘too much’, and ‘a lot more’.</p> <p>When comparing lengths directly, children need to ensure that they align the starting points, and compare like-for-like, e.g. straightening skipping ropes before comparing lengths.</p> <p>When comparing capacities directly, children can pour from one container to another to find which holds more, or find one that is the same. However, children may conclude that if one container overflows that must mean ‘bigger’. Ensure that children have opportunities to see a jug of coloured water poured into a range of container shapes. Ask: ‘What do you think will happen if we pour this tall thin jugful into this short fat dish?’</p>	<ul style="list-style-type: none"> <li>● encouraging children to compare different attributes in everyday situations: ‘I wonder who has the longest snake?’ ‘I wonder whose pot will hold the most water?’ ‘I wonder which ball is the heaviest?’</li> <li>● cutting a piece of ribbon as long as a child’s arm and encouraging them to find things in the environment that are longer, shorter or the same length</li> <li>● focusing on asking for specific things according to their attributes. For example: ‘Please can you pass me a ... that is ... than this one?’</li> <li>● when comparing directly, finding the odd one out, by providing a varied range of container shapes all containing the same amount of liquid except for one. ‘Which one do you think is the odd one out? Why? How will we check? Were we right?’</li> <li>● posing see-saw problems, relating to weight: ‘What can we do to make this side of the see-saw go down?’</li> <li>● using a simple spring balance to compare the weight of cargo for a toy boat</li> </ul>



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<p>Comparing weight can be tricky to conceptualise. One way is to identify that greater mass is shown by a greater downward pull. Ask children to hold a carrier bag; encourage them to notice it feels as though their hand is being pulled down when something heavy is put in it. Place a carrier bag in each hand and identify which one is heavier, by discussing which arm feels more pulled down. Show this using a simple spring balance or a box attached to elastic bands; identify that the elastic is being stretched by being pulled down, just like our arms.</p> <p>Explore the link to the balance scales to show that the heavier side goes down. If possible, exemplify this with a see-saw.</p> <p>Ensure that children are presented with large, light things and small, heavy things, to prevent the over-generalisation that big means heavy and small means light.</p>	<ul style="list-style-type: none"> <li>● setting up a 'balancing station' with interesting things to weigh and to balance, indoors and outdoors</li> <li>● comparing different parcels, ensuring some of the smaller parcels are heavy, and some of the larger parcels are light.</li> </ul>
<b>Showing awareness of comparison in estimating and predicting</b>	<b>Activities &amp; opportunities</b>
<p>After children have had lots of practical experiences of comparing attributes, they can begin to estimate and to predict. For instance, they can start to consider which container would be best to store a specific item in: 'Which box should Teddy have?', 'What will fit in here?'</p>	<ul style="list-style-type: none"> <li>● making a bed for a teddy using blocks</li> <li>● selecting a box or container to store a specific item</li> <li>● dressing dolls, and selecting different-sized clothes</li> <li>● finding things that will fit inside a matchbox.</li> </ul>
<b>Comparing indirectly</b>	<b>Activities &amp; opportunities</b>
<p>Children can then move onto using one thing to compare with two others, if, for example, asked to put things in order of height, weight or capacity. This may involve using a 'go between', for instance pouring a jugful of water into two bottles to see which holds more. Problems may be posed such as: 'I would like to move this table outside – do you think it will fit through the door?'</p>	<ul style="list-style-type: none"> <li>● making 'Russian doll'-type sets of nesting boxes from a collection</li> <li>● finding ways of seeing if the cupboard or carpet will fit in the role-play area without moving it</li> <li>● finding which of three pairs of shoes is heaviest for packing in a rucksack</li> <li>● packing a shopping bag, making sure the lightest items do not get squashed by heavier things.</li> </ul>
<b>Recognising the relationship between the size and number of units</b>	<b>Activities &amp; opportunities</b>
<p>Before children use standard units of measure, they begin to compare units of different sizes in practical contexts. One example may be in the water tray, where children realise it will take them longer to fill a bucket using teaspoons than bottles. Another example would be to fill identical containers with different-sized objects, e.g. small balls or large balls. These sorts of playful experiences enable children to make the generalisation that the smaller the unit the more we need of them, or the bigger the unit the less we need of them. These experiences can be extended by encouraging estimations: 'How many tennis balls do you think will fit in this tub?' Then check this by filling it. 'What if I try to fill it with ping pong balls? Will our answer stay the same? If not, why not?'</p>	<ul style="list-style-type: none"> <li>● setting up an Estimation Station and guessing how many things are in the jar each day</li> <li>● making biscuits from a given amount of dough – choosing cutters to see who will make the most biscuits</li> <li>● choosing from a selection of spoons, ladles, etc, to see who can fill their pot the quickest with rice. How do you know who will be quickest?</li> </ul>

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<p>In practical situations, these sorts of questions can be asked to support children in their justification of the choice of equipment. For example: ‘What can I use to help fill the water tray? Which bag shall I use for my shopping? Which box would be best to store these buttons? Why did you think that is a good choice?’</p>	
<p><b>Beginning to use units to compare things</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Experiences can be provided where children use units to ‘measure’ and compare. It is better to provide identical bricks, centimetre cubes or metre sticks so they can count physical units, rather than repeating the use of one item as with using hands or feet. In order to measure accurately, they need to ensure there are no gaps between units of measure. Using standard units helps children make connections with measuring in ‘real life’. Young children also enjoy using height charts, measuring tapes, rulers, digital scales and timers, although will not yet fully understand how they work.</p>	<ul style="list-style-type: none"> <li>● setting up a ‘filling station’ with lots of different-sized containers to fill with beads, then comparing capacities</li> <li>● using large bricks to measure the height of individuals</li> <li>● using metre sticks to see if an elephant or dinosaur would fit in the room</li> <li>● measuring the growth of a beanstalk or sunflower with interlocking centimetre cubes</li> <li>● comparing the capacity of different bottles by filling lots of glasses.</li> </ul>
<p><b>Beginning to use time to sequence events</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Time is an abstract aspect to measure, and tricky in a range of ways. Although their age may be the most familiar number they know, children may have little sense of the unit of a ‘year’, and few may know the date of their birthday.</p> <p>In order to tell the time, children need a sense of number, space and time, the ability to count, and some notion of fractions (for half and quarter hours). In the Early Years we begin by drawing children’s attention to sequencing of activities, important times in their day, and some sequences of time that are significant to them.</p> <p>Vocabulary that supports the understanding of this concept includes the positional language of ‘before’, ‘after’, ‘next’, and the relative terms ‘yesterday’ and ‘tomorrow’. Knowing days of the week also helps children to keep track of time. Direct children’s attention to the short hand, pointing to a number on a clockface, and identify what we are doing at that time.</p>	<ul style="list-style-type: none"> <li>● un-muddling visual timetables</li> <li>● making picture sequences for cooking instructions</li> <li>● describing sequences by re-telling stories</li> <li>● discussing ‘o’clock’ times at registration, lunchtime, snack time, tidy-up time, etc.</li> <li>● making their own timetable for a day – selecting activities and ordering them.</li> </ul>
<p><b>Beginning to experience specific time durations</b></p>	<p><b>Activities &amp; opportunities</b></p>
<p>Children need to experience specific time spans in order to start to develop an overall sense of time. Initially, this may be based on familiar activities such as the number of ‘sleeps’ before an event. A class calendar may support this by highlighting certain events (‘How many sleeps until the chicks start to hatch?’, ‘How many sleeps until my birthday?’, ‘How many sleeps until we go to the park?’). Discuss the number of sleeps getting smaller and what this means.</p> <p>By using timers in play, children can start to explore what they can do in a certain time period. For example: ‘I wonder how long it takes you to run around the track?’, ‘How</p>	<ul style="list-style-type: none"> <li>● events on a class calendar to count down to</li> <li>● timers provided for children to set and respond to challenges; e.g. ‘I wonder if we can run as fast as a cheetah’, ‘I wonder how many hops I can do in ten seconds’, ‘I wonder how many times I can write my name in a minute’, etc.</li> <li>● time durations with songs or music.</li> </ul>

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would we know if you were getting quicker?'. Identify that, in this case, the smaller the number of seconds the quicker you are getting (this is tricky for a child, as usually bigger numbers are 'better').

Children may also have the opportunity to see how many things they can do in a minute. For example: 'How many play people can you rescue from the pit?' (Wrap fabric around a water tray to create small gaps through which people can be rescued.)

### Common errors in this area:

- keeping track of events, e.g. 'Have I had my lunch yet?'
- positional language associated with time; muddling the relative terms 'yesterday' and 'tomorrow'
- using 'long' to describe the shape of something (e.g. a block that is much longer than it is wide) rather than to compare lengths
- not taking into account both ends as the starting and stopping point
- not being able to say 'than' in the phrase, 'this is longer than that'
- not understanding that units must cover a complete length, with no gaps or overlaps, demonstrated by thinking that measuring is about counting units placed along something, or putting a ruler alongside and saying a number
- not understanding that units must be equal.

### What to look for, can a child:

- find something that is longer, shorter, heavier, lighter (etc.) than a reference item?
- find an appropriate container for a specific item?
- describe the location of something using positional language?
- accurately use the relative terms 'yesterday' and 'tomorrow'?
- order a short sequence of events?